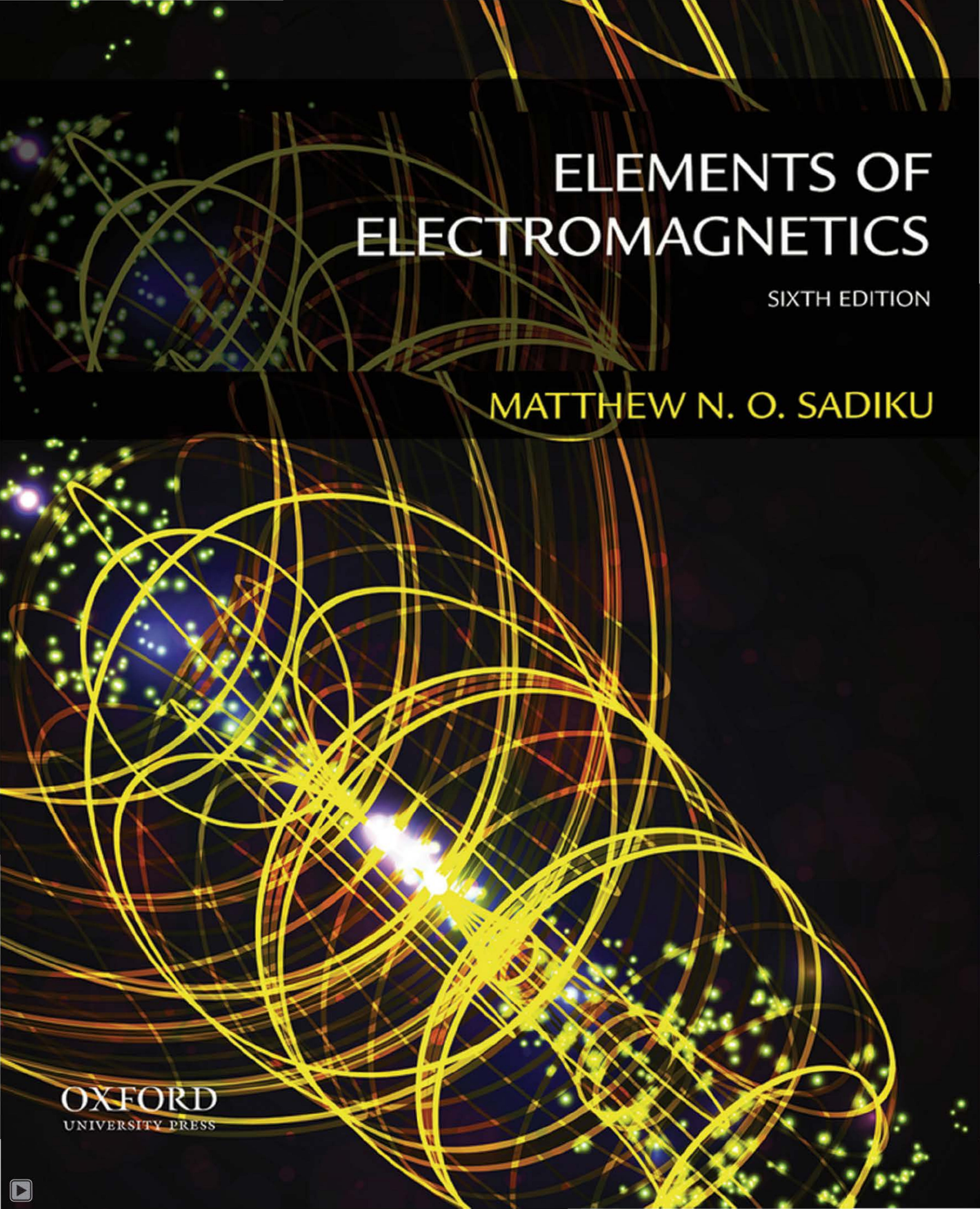

ELEMENTS OF ELECTROMAGNETICS

SIXTH EDITION

MATTHEW N. O. SADIKU

OXFORD
UNIVERSITY PRESS

ELEMENTS OF ELECTROMAGNETICS

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Published in the United States of America by
Oxford University Press
198 Madison Avenue, New York, NY 10016
http://www.oup.com

Library of Congress Cataloging-in-Publication Data

Sadiku, Matthew N. O., author.
Elements of electromagnetics / Matthew N.O. Sadiku, Prairie View A&M University. – Sixth edition.
pages cm.—(The Oxford series in electrical and computer engineering)
Includes bibliographical references and index.
ISBN 978-0-19-932138-4
1. Electromagnetism. I. Title.
QC760.S23 2015
537—dc23

2013033865

Printing number: 9 8 7 6 5 4 3 2 1
Printed in the United States of America
on acid-free paper

To my wife, Kikelomo

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† Indicates sections that may be skipped, explained briefly, or assigned as homework if the text is covered in one semester.

PART 4: WAVES AND APPLICATIONS

PREFACE

Each revision of this book has embodied many changes that make the book better. However, the core of the subject of electromagnetics never changes. The fundamental objective of the book remains the same as in the first edition: to present electromagnetics concepts in a clearer and more interesting manner than other texts do. This objective is achieved in the following ways:

1. In order to avoid complicating matters by covering electromagnetic and mathematical concepts simultaneously, vector analysis is covered at the beginning of the text and applied gradually. This approach avoids breaking in repeatedly with more background on vector analysis, thereby creating discontinuity in the flow of thought. It also separates mathematical theorems from physical concepts and makes it easier for the student to grasp the generality of those theorems.

2. Each chapter opens either with a historical profile of some electromagnetic pioneers or a discussion of a modern topic related to the chapter. The chapter starts with a brief introduction that serves as a guide to the whole chapter and also links the chapter to the rest of the book. The introduction helps the students see the need for the chapter and how it relates to the previous chapter. Key points are emphasized to draw the readers' attention. A brief summary of the major concepts is discussed toward the end of the chapter.

3. To ensure that students clearly understand the subject matter, key terms are defined and highlighted. Important formulas are boxed to help students identify essential formulas.

4. Each chapter includes a reasonable number of solved examples. Because the examples are part of the text, they are clearly explained without asking the readers to fill in missing steps. In writing out the solution, we aim for clarity rather than efficiency. Thoroughly worked-out examples give students confidence to solve problems themselves and to learn to apply concepts, which is an integral part of engineering education. Each illustrative example is followed by a problem in the form of a Practice Exercise, with the answer provided.

5. At the end of each chapter are 10 review questions in the form of multiple-choice objective items. It has been found that open-ended questions, although intended to be thought provoking, are ignored by most students. Objective review questions with answers immediately following them provide encouragement for students to do the problems and gain immediate feedback. A large number of problems are provided and are presented in the same order as the material in the main text. Problems of intermediate difficulty are identified by a single asterisk (*); the most difficult problems are marked with a double asterisk (**). Enough problems are provided to allow the instructor to choose some as examples and assign some as homework problems. Answers to odd-numbered problems are provided in Appendix E.

6. Because most practical applications involve time-varying fields, six chapters are devoted to such fields. However, static fields are given proper emphasis because they are special cases of dynamic fields. Ignorance of electrostatics is no longer acceptable

A NOTE TO THE STUDENT

Electromagnetic theory is generally regarded by students as one of the most difficult courses in physics or the electrical engineering curriculum. But this misconception may be proved wrong if you take some precautions. From experience, the following ideas are provided to help you perform to the best of your ability with the aid of this textbook:

1. Pay particular attention to Part 1 on vector analysis, the mathematical tool for this course. Without a clear understanding of this section, you may have problems with the rest of the book.
2. Do not attempt to memorize too many formulas. Memorize only the basic ones, which are usually boxed, and try to derive others from these. Try to understand how formulas are related. Obviously, there is nothing like a general formula for solving all problems. Each formula has some limitations owing to the assumptions made in obtaining it. Be aware of those assumptions and use the formula accordingly.
3. Try to identify the key words or terms in a given definition or law. Knowing the meaning of these key words is essential for proper application of the definition or law.
4. Attempt to solve as many problems as you can. Practice is the best way to gain skill. The best way to understand the formulas and assimilate the material is by solving problems. It is recommended that you solve at least the problems in the Practice Exercise immediately following each illustrative example. Sketch a diagram illustrating the problem before attempting to solve it mathematically. Sketching the diagram not only makes the problem easier to solve, it also helps you understand the problem by simplifying and organizing your thinking process. Note that unless otherwise stated, all distances are in meters. For example (2, −1, 5) actually means (2 m, −1 m, 5 m).

You may use MATLAB to do number crunching and plotting. A brief introduction to MATLAB is provided in Appendix C.

A list of the powers of 10 and Greek letters commonly used throughout this text is provided in the tables located on the inside cover. Important formulas in calculus, vectors, and complex analysis are provided in Appendix A. Answers to odd-numbered problems are in Appendix E.

MATH ASSESSMENT

PART 1: VECTOR ANALYSIS

1.1 Find the value of the determinant $V = \begin{vmatrix} 2 & 5 & 4 \\ 4 & 1 & 6 \\ 3 & 0 & 1 \end{vmatrix}$.

1.2 A 12 m ladder is made to lean on a wall making an angle of 30° with the horizontal. Find the distance of its base from the wall and also the height at the point where it touches the wall.

1.3 What is the distance R between the two points $A(3, 5, 1)$ and $B(5, 7, 2)$? Also find its reciprocal, $\frac{1}{R}$.

1.4 What is the distance vector $\mathbf{R}_{AB}$ from $A(3, 7, 1)$ to $B(8, 19, 2)$ and a unit vector $\mathbf{a}_{AB}$ in the direction of $\mathbf{R}_{AB}$?

1.5 Given $\mathbf{A} = 2\mathbf{a}_x - 4\mathbf{a}_y + 3\mathbf{a}_z$ and $\mathbf{B} = \mathbf{a}_x + 2\mathbf{a}_y + 5\mathbf{a}_z$, find a unit vector $\mathbf{a}_C$ that is perpendicular to both $\mathbf{A}$ and $\mathbf{B}$.

1.6 There are four charges in space at four points A, B, C, and D, each 1m from *every* other. You are asked to make a selection of coordinates for these charges. How do you place them in space and select their coordinates? There is no unique way.

1.7 A man driving a car starts at point O, drives in the following pattern

15 km northeast to point A,
20 km southwest to a point B,
25 km north to C,
10 km southeast to D,
15 km west to E, and stops.

How far is he from his starting point, and in what direction?

1.8 A unit vector $\mathbf{a}_n$ makes angles α, β, and γ with the x-, y-, and z-axes, respectively. Express $\mathbf{a}_n$ in the rectangular coordinate system. Also express a nonunit vector $\overrightarrow{OP}$ of length ℓ parallel to $\mathbf{a}_n$.

1.9 A vector $\overrightarrow{OP}$ makes angles 75.5225° and 64.3411° with the x- and y-axes, respectively. Another vector $\overrightarrow{OQ}$ makes 52.2388° with both x- and y-axes equally. Find the angle between $\overrightarrow{OP}$ and $\overrightarrow{OQ}$.

1.10 An experiment revealed that the point $Q(x', y', z')$ is 4 m from $P(2, 1, 4)$ and that the vector $\overrightarrow{QP}$ makes 45.5225°, 59.4003°, and 60° with the x-, y-, and z-axes, respectively. Determine the location of Q.

1.11 In a certain frame of reference with x-, y-, and z-axes, imagine the first octant to be a room with a door. Suppose that the height of the door is h and its width is ρ. The top-right corner P of the door when it is shut has the rectangular coordinates $(\rho, 0, h)$. Now if the door is turned by angle ϕ, so we can enter the room, what are the coordinates of P? What is the length of its diagonal $r = \overline{OP}$ in terms of ρ and z? Suppose the vector $\overrightarrow{OP}$ makes an angle θ with the z-axis; express ρ and h in terms of r and θ.

1.12 Consider an ellipse in the xy-plane described by the equation $x^2 - xy + y^2 = 7$. Find a unit normal to the curve (i) at a general point $P(x, y)$ and (ii) at $(-1, 2)$ in particular.

1.13 What is the distance from origin to the plane $2x + 2y - z = 15$? (*Hint*: The unit normal to any surface $f(x, y, z) = c$ is given by $\mathbf{a}_n = \frac{\nabla f}{|\nabla f|}$. The distance from origin to a given plane then is the dot product $\mathbf{a}_n \cdot \mathbf{r}$, where $\mathbf{r}$ is a vector from the origin to any point on the plane; that is, it is a position vector of a point on the plane.

1.14 Given $f = x^2y + 3z + 4$, find (i) ∇f, (ii) $\nabla^2 f$, and (ii) a unit normal to $f = 0$ at $(1, -2, 4)$.

1.15 Given $\mathbf{A} = 2xy\,\mathbf{a}_x + 3zy\,\mathbf{a}_y + 5z\,\mathbf{a}_z$ and $\mathbf{B} = \sin x\,\mathbf{a}_x + 2y\,\mathbf{a}_y + 5y\,\mathbf{a}_z$, find (i) $\nabla \cdot \mathbf{A}$, (ii) $\nabla \times \mathbf{A}$, (iii) $\nabla \cdot \nabla \times \mathbf{A}$, and (iv) $\nabla \cdot (\mathbf{A} \times \mathbf{B})$.

PART 2: ELECTROSTATICS

2.1 Define even and odd functions. Show that

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(x) \text{ is even} \\ 0, & \text{if } f(x) \text{ is odd} \end{cases}$$

2.2 Evaluate the integral $\int_{-L}^{L} \frac{\rho \mathbf{a}_\rho - z\mathbf{a}_z}{(\rho^2 + z^2)^{3/2}}\,dz$.

2.3 The cross section of a cylinder of radius 3 is shown in Figure MA-1. A straight charged wire MM' is found to pass through it at an angle θ with the diameter as shown. Find the length of the wire $\overline{OF}$ enclosed if $\theta = \pi/6$.

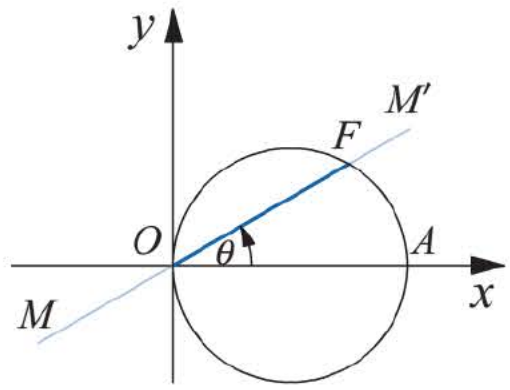

FIGURE MA-1 For Problem 2.3

2.4 Show that the ordinary angle subtended by a closed curve lying in a plane at a point P is 2π radians if P is enclosed by the curve and zero if not.

2.5 Show that the solid angle subtended by a closed surface at a point P is 4π steradians if P is enclosed by the closed surface and zero if not.

2.6 You are given a vector function **A** and a closed surface S. Determine the divergence of **A** and verify both sides of the divergence theorem on S.

$$\mathbf{A} = 2xy\,\mathbf{a}_x + 3zy\,\mathbf{a}_y + 5z\,\mathbf{a}_z \qquad S: -2 \le x \le 3,\ 4 \le y \le 6,\ -3 \le z \le 4$$

2.7 You are given a vector function **A** and a closed surface S. Determine the divergence of **A** and verify both sides of the divergence theorem on S.

$$\mathbf{A} = 3\rho \sin\phi\ \mathrm{a}\rho + 6\rho^2\,\mathbf{a}\phi + 5z\,\mathbf{a}_z \qquad S: 3 \le \rho \le 5,\ \frac{\pi}{6} \le \phi \le \frac{\pi}{3},\ 2 \le z \le 5$$

2.8 You are given a vector function **A** and a closed surface S. Determine the divergence of **A** and verify both sides of the divergence theorem on S.

$$\mathbf{A} = 2r^2\,\mathbf{a}_r + 5r\sin\theta\cos\phi\,\mathbf{a}_\theta + 4\cos\theta\,\mathbf{a}_\phi \quad S: 3 \le r \le 5,\ \frac{\pi}{6} \le \theta \le \frac{\pi}{3},\ 0 \le \phi \le \frac{\pi}{4}$$

2.9 A sphere of radius a centered at the origin has a total charge of 20 C uniformly distributed over its entire volume. If all the charge above the plane $z = h$ is removed, how much charge is left in the sphere? Take $a = 10\text{m}$ and $h = 6\text{m}$.

2.10 Consider a vector given by $\mathbf{A} = (4xy + z)\mathbf{a}_x + 2x^2\mathbf{a}_y + x\mathbf{a}_z$. Find the line integral from $A(3, 7, 1)$ to $B(8, 9, 2)$ by (i) evaluating the line integral $V_{AB} = -\int_A^B \mathbf{E} \cdot \mathbf{dl}$ along the line joining A to B and (ii) evaluating $\left\{-\int_A^C \mathbf{E} \cdot \mathbf{dl} - \int_C^D \mathbf{E} \cdot \mathbf{dl} - \int_D^B \mathbf{E} \cdot \mathbf{dl}\right\}$, where the stopovers C and D are $C(8, 7, 1)$ and $D(8, 9, 1)$.

2.11 Given a vector $\mathbf{A} = 3\mathbf{a}_x - 2\mathbf{a}_y - 16\mathbf{a}_z$, find its components normal to the plane $2x + 2y - z + 8 = 0$ and tangential to it.

2.12 At the interface $y = 0$ a certain vector field is given by

$$\mathbf{A} = \begin{cases} e^{-2y}\mathbf{a}_x + 5\mathbf{a}_y & \text{for} \quad y > 0 \\ e^{2y}\mathbf{a}_x + 3\mathbf{a}_y & \text{for} \quad y < 0 \end{cases}$$

Taking the tangential and normal components, state which is continuous and which is not.

2.13 You are given the plane S with a unit normal $\mathbf{a}_n$ and passing through M. Describe the method for finding the image P' of a point P in S for the particular case in which $\mathbf{a}_n = \dfrac{3\mathbf{a}_x - 2\mathbf{a}_y + 6\mathbf{a}_z}{\sqrt{3^2 + 2^2 + 6^2}}$ and S passes through $M(1, 0, 3)$. Find the coordinates of the image of $P(2, 1, 5)$ in S.

PART 3: MAGNETOSTATICS

3.1 A slant wire in the *xy*-plane joining the points $O(0, 0)$ and $P(6, 8)$ carries a current of 5 A from O to P as shown in Figure MA-2. Find the quantities $\cos\alpha_1$ and $\cos\alpha_2$ and the distance $\overline{MN}$.

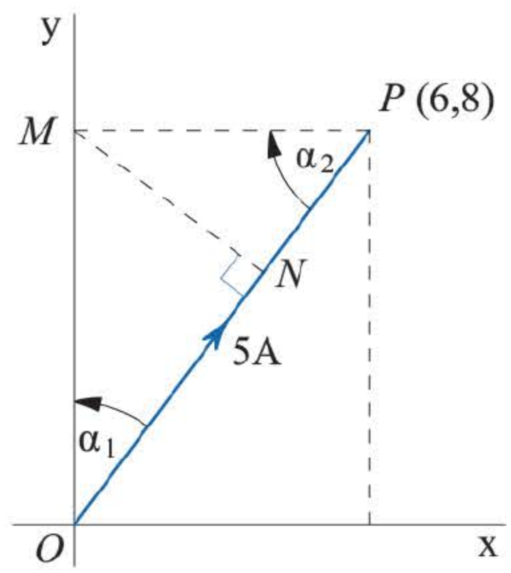

FIGURE MA-2 A slant wire for Problem 3.1.

3.2 You are given a vector function **A** and an open surface S bounded by C. Determine the curl of each vector function and verify both sides of Stokes's theorem on S and along C.

$$\mathbf{A} = 2xy\,\mathbf{a}_x + 3zy\,\mathbf{a}_y + 5z\,\mathbf{a}_z \qquad S: -2 \le x \le 3,\ 4 \le y \le 6,\ z = 4$$

3.3 You are given a vector function **A** and an open surface S bounded by C. Determine the curl of each vector function and verify both sides of Stokes's theorem on S and along C.

$$\mathbf{A} = 3\rho\sin\phi\,\mathbf{a}_\rho + 6\rho^2\,\mathbf{a}_\phi + 5z\,\mathbf{a}_z \qquad S: 3 \le \rho \le 5,\ \frac{\pi}{6} \le \phi \le \frac{\pi}{3},\ z \le 7$$

3.4 You are given a vector function **A** and an open surface S bounded by C. Determine the curl of each vector function and verify both sides of Stokes's theorem on S and along C.

$$\mathbf{A} = 2r^2\,\mathbf{a}_r + 5r\sin\theta\cos\phi\,\mathbf{a}_\theta + 4\cos\theta\,\mathbf{a}_\phi \qquad S: r = 5,\ \frac{\pi}{6} \le \theta \le \frac{\pi}{3},\ 0 \le \phi \le \frac{\pi}{4}$$

PART 4: WAVES AND APPLICATIONS

4.1 Show that $\nabla \cdot \nabla \times \mathbf{A} = 0$.

4.2 Show that $\nabla \times \nabla \psi = \mathbf{0}$.

4.3 Show that $\nabla \times \nabla \times \mathbf{A} = \nabla\nabla \cdot \mathbf{A} - \nabla^2\mathbf{A}$.

4.4 Show that $\nabla \cdot \mathbf{A} \times \mathbf{B} = \mathbf{B} \cdot \nabla \times \mathbf{A} - \mathbf{A} \cdot \nabla \times \mathbf{B}$.

4.5 Use *De Moivre's theorem* to prove that $\cos 3\theta = \cos^3 \theta - 3 \cos \theta \sin^2\theta$:

4.6 Determine $\sqrt{j}$.

4.7 Determine $\sqrt{j}$ using the Euler formula.

4.8 Find the phasors for the following field quantities:

(a) $E_x(z, t) = E_o \cos(\omega t - \beta z + \phi)$ (V/m)

(b) $E_y(z, t) = 100e^{-3z} \cos(\omega t - 5z + \pi/4)$ (V/m)

(c) $H_x(z, t) = H_o \cos(\omega t + \beta z)$ (A/m)

(d) $H_y(z, t) = 120\pi e^{-5z} \cos(\omega t + \beta z + \phi_h)$ (A/m)

4.9 Find the instantaneous time domain sinusoidal functions corresponding to the following phasors:

(a) $E_x(z) = E_o e^{j\beta z}$ (V/m)

(b) $E_y(z) = 100e^{-3z}e^{-j5z}$ (V/m)

(c) $I_s(z) = 5 + j4$ (A)

(d) $V_s(z) = j10e^{j\pi/3}$ (V)

4.10 Write the phasor expression $\tilde{I}$ for the following current using a cosine reference.

(a) $i(t) = I_o \cos(\omega t - \pi/6)$

(b) $i(t) = I_o \sin(\omega t + \pi/3)$

4.11 Find the instantaneous $V(t)$ for the following phasors using a cosine reference.

(a) $\tilde{V}_s = V_o e^{j\pi/4}$

(b) $\tilde{V}_s = 12 - j5$ (V)

4.12 The unit normal to a plane is given by $\mathbf{a}_n$. It also passes through a point S whose position vector is $\mathbf{r}_o$. Determine the equation of the plane.

4.13 A voltage source $V(t) = 100 \cos(6\pi 10^9 t - 45°)$ (V) is connected to a series *RLC* circuit, as shown in Figure MA-3. Given $R = 10\,\text{M}\Omega$, $C = 100\,\text{pF}$, and $L = 1\,\text{H}$, use phasor notation to find the following:

(a) $i(t)$

(b) $V_c(t)$, the voltage across the capacitor

4.14 (i) Show that the locus of the points $P(x, y)$ obeying the equation

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

represents a circle. (ii) Express the coordinates of the center and the radius. Use the following equations of circles to find the centers and radii.

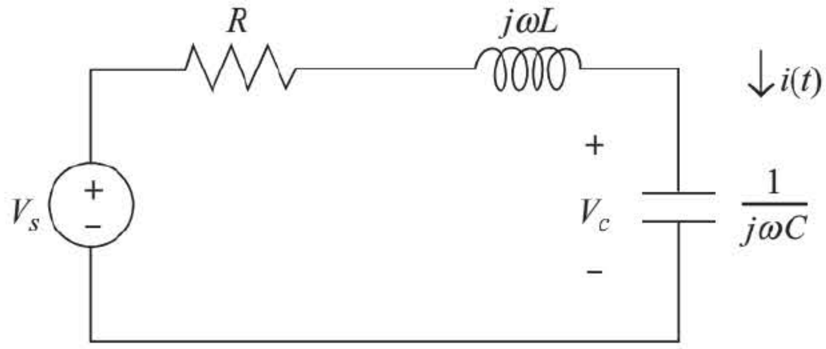

FIGURE MA-3 A series *RLC* circuit for Problem 4.13.

$$x^2 + y^2 + 8x - 4y + 11 = 0$$
$$x^2 + y^2 - 10x + 6y + 9 = 0$$
$$225x^2 + 225y^2 + 90x - 300y + 28 = 0$$

4.15 Recall the vector identity $\nabla \times \psi\mathbf{A} \equiv \psi\nabla \times \mathbf{A} + \nabla\psi \times \mathbf{A}$, where ψ is a scalar function and $\mathbf{A}$ is a vector point function. Suppose $\mathbf{A} = A_z\mathbf{a}_z$, where $A_z = \frac{e^{-jkr}}{r}$ and k is a constant. Simplify $\nabla \times \mathbf{A}$.

4.16 Show that the sum of the first N terms in a geometric series $S_N = 1 + x + x^2 + \cdots + x^{N-1} = \frac{1 - x^N}{1 - x}$ and further assuming $|x| < 1$, show that:

$$\frac{1}{1 - x} = 1 + x + x^2 + x^3 + \cdots$$

$$\frac{1}{1 + x} = 1 - x + x^2 - x^3 + - \cdots$$

4.17 Show the following series expansion assuming $|x| < 1$:

$$\frac{1}{(1 - x)^2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$$

4.18 From the binomial expansion of $(a + b)^n$, build the Pascal triangle for $n = 0, 1, 2, \cdots, 5$.

PART

1

VECTOR ANALYSIS

CODES OF ETHICS

Engineering is a profession that makes significant contributions to the economic and social well-being of people all over the world. As members of this important profession, engineers are expected to exhibit the highest standards of honesty and integrity. Unfortunately, the engineering curriculum is so crowded that there is no room for a course on ethics in most schools. Although there are over 850 codes of ethics for different professions all over the world, the code of ethics of the Institute of Electrical and Electronics Engineers (IEEE) is presented here to give students a flavor of the importance of ethics in engineering professions.

We, the members of the IEEE, in recognition of the importance of our technologies in affecting the quality of life throughout the world, and in accepting a personal obligation to our profession, its members and the communities we serve, do hereby commit ourselves to the highest ethical and professional conduct and agree:

1. to accept responsibility in making engineering decisions consistent with the safety, health, and welfare of the public, and to disclose promptly factors that might endanger the public or the environment;
2. to avoid real or perceived conflicts of interest whenever possible, and to disclose them to affected parties when they do exist;
3. to be honest and realistic in stating claims or estimates based on available data;
4. to reject bribery in all its forms;
5. to improve the understanding of technology, its appropriate application, and potential consequences;
6. to maintain and improve our technical competence and to undertake technological tasks for others only if qualified by training or experience, or after full disclosure of pertinent limitations;
7. to seek, accept, and offer honest criticism of technical work, to acknowledge and correct errors, and to credit properly the contributions of others;
8. to treat fairly all persons regardless of such factors as race, religion, gender, disability, age, or national origin;
9. to avoid injuring others, their property, reputation, or employment by false or malicious action;
10. to assist colleagues and co-workers in their professional development and to support them in following this code of ethics.

—Courtesy of IEEE

CHAPTER 1

VECTOR ALGEBRA

One machine can do the work of fifty ordinary men. No machine can do the work of one extraordinary man.

—ELBERT HUBBARD

1.1 INTRODUCTION

Electromagnetics (EM) may be regarded as the study of the interactions between electric charges at rest and in motion. It entails the analysis, synthesis, physical interpretation, and application of electric and magnetic fields.

Electromagnetics (EM) is a branch of physics or electrical engineering in which electric and magnetic phenomena are studied.

EM principles find applications in various allied disciplines such as microwaves, antennas, electric machines, satellite communications, bioelectromagnetics, plasmas, nuclear research, fiber optics, electromagnetic interference and compatibility, electromechanical energy conversion, radar meteorology, and remote sensing.[1,2] In physical medicine, for example, EM power, in the form either of shortwaves or microwaves, is used to heat deep tissues and to stimulate certain physiological responses in order to relieve certain pathological conditions. EM fields are used in induction heaters for melting, forging, annealing, surface hardening, and soldering operations. Dielectric heating equipment uses shortwaves to join or seal thin sheets of plastic materials. EM energy offers many new and exciting possibilities in agriculture. It is used, for example, to change vegetable taste by reducing acidity.

EM devices include transformers, electric relays, radio/TV, telephones, electric motors, transmission lines, waveguides, antennas, optical fibers, radars, and lasers. The design of these devices requires thorough knowledge of the laws and principles of EM.

[1]For numerous applications of electrostatics, see J. M. Crowley, *Fundamentals of Applied Electrostatics*. New York: John Wiley & Sons, 1986.

[2]For other areas of applications of EM, see, for example, D. Teplitz, ed., *Electromagnetism: Paths to Research*. New York: Plenum Press, 1982.

†1.2 A PREVIEW OF THE BOOK

The subject of electromagnetic phenomena in this book can be summarized in Maxwell's equations:

$$\nabla \cdot \mathbf{D} = \rho_v \tag{1.1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{1.2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{1.3}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \tag{1.4}$$

where ∇ = the vector differential operator
$\mathbf{D}$ = the electric flux density
$\mathbf{B}$ = the magnetic flux density
$\mathbf{E}$ = the electric field intensity
$\mathbf{H}$ = the magnetic field intensity
ρ_v = the volume charge density
$\mathbf{J}$ = the current density

Maxwell based these equations on previously known results, both experimental and theoretical. A quick look at these equations shows that we shall be dealing with vector quantities. It is consequently logical that we spend some time in Part 1 examining the mathematical tools required for this course. The derivation of eqs. (1.1) to (1.4) for time-invariant conditions and the physical significance of the quantities **D**, **B**, **E**, **H**, **J**, and ρ_v will be our aim in Parts 2 and 3. In Part 4, we shall reexamine the equations for time-varying situations and apply them in our study of practical EM devices.

1.3 SCALARS AND VECTORS

Vector analysis is a mathematical tool with which electromagnetic concepts are most conveniently expressed and best comprehended. We must learn its rules and techniques before we can confidently apply it. Since most students taking this course have little exposure to vector analysis, considerable attention is given to it in this and the next two chapters.[3] This chapter introduces the basic concepts of vector algebra in Cartesian coordinates only. The next chapter builds on this and extends to other coordinate systems.

A quantity can be either a scalar or a vector.

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†Indicates sections that may be skipped, explained briefly, or assigned as homework if the text is covered in one semester.

[3]The reader who feels no need for review of vector algebra can skip to the next chapter.

A **scalar** is a quantity that has only magnitude.

Quantities such as time, mass, distance, temperature, entropy, electric potential, and population are scalars.

A **vector** is a quantity that has both magnitude and direction.

Vector quantities include velocity, force, displacement, and electric field intensity. Another class of physical quantities is called *tensors,* of which scalars and vectors are special cases. For most of the time, we shall be concerned with scalars and vectors.[4]

To distinguish between a scalar and a vector it is customary to represent a vector by a letter with an arrow on top of it, such as $\vec{A}$ and $\vec{B}$, or by a letter in boldface type such as **A** and **B**. A scalar is represented simply by a letter—for example, A, B, U, and V.

EM theory is essentially a study of some particular fields.

A **field** is a function that specifies a particular quantity everywhere in a region.

If the quantity is scalar (or vector), the field is said to be a scalar (or vector) field. Examples of scalar fields are temperature distribution in a building, sound intensity in a theater, electric potential in a region, and refractive index of a stratified medium. The gravitational force on a body in space and the velocity of raindrops in the atmosphere are examples of vector fields.

1.4 UNIT VECTOR

A vector **A** has both magnitude and direction. The *magnitude* of **A** is a scalar written as A or $|\mathbf{A}|$. A *unit vector* $\mathbf{a}_A$ along **A** is defined as a vector whose magnitude is unity (i.e., 1) and its direction is along **A**, that is,

$$\boxed{\mathbf{a}_A = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\mathbf{A}}{A}} \tag{1.5}$$

Note that $|\mathbf{a}_A| = 1$. Thus we may write **A** as

$$\mathbf{A} = A\mathbf{a}_A \tag{1.6}$$

which completely specifies **A** in terms of its magnitude A and its direction $\mathbf{a}_A$.

A vector **A** in Cartesian (or rectangular) coordinates may be represented as

$$(A_x, A_y, A_z) \qquad \text{or} \qquad A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z \tag{1.7}$$

[4] For an elementary treatment of tensors, see, for example, A. I. Borisenko and I. E. Tarapor, *Vector and Tensor Analysis with Applications.* New York: Dover, 1979.

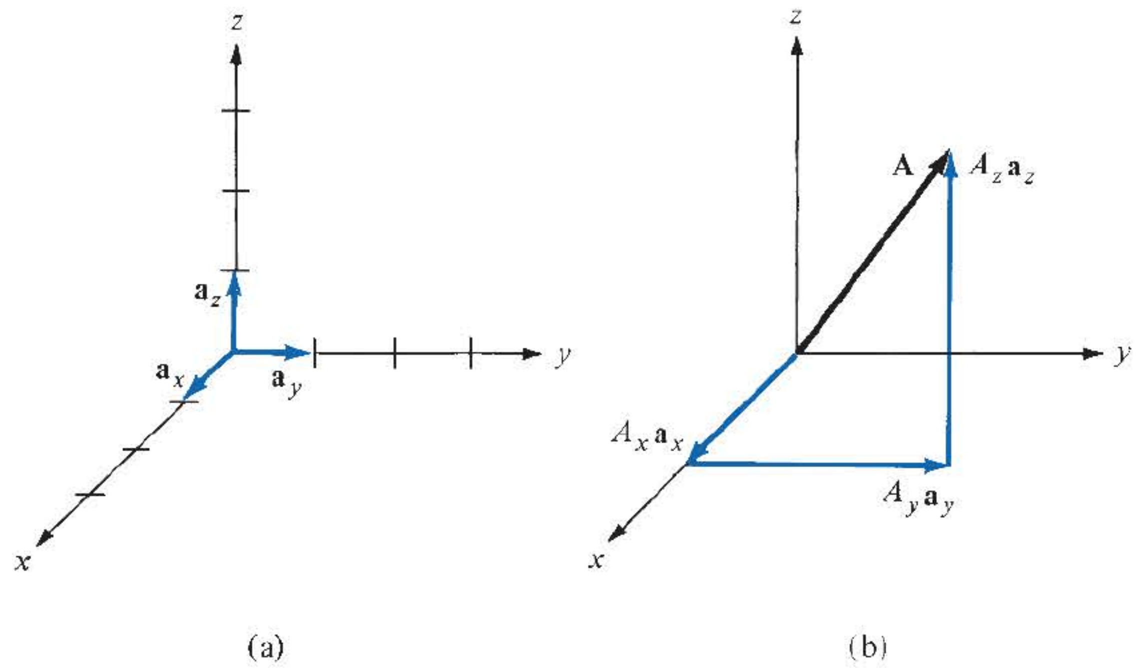

FIGURE 1.1 (**a**) Unit vectors $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$, (**b**) components of **A** along $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$.

where A_x, A_y, and A_z are called the *components of* **A** in the x-, y-, and z-directions, respectively; $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$ are unit vectors in the x-, y-, and z-directions, respectively. For example, $\mathbf{a}_x$ is a dimensionless vector of magnitude one in the direction of the increase of the x-axis. The unit vectors $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$ are illustrated in Figure 1.1(a), and the components of **A** along the coordinate axes are shown in Figure 1.1(b). The magnitude of vector **A** is given by

$$\boxed{A = \sqrt{A_x^2 + A_y^2 + A_z^2}} \tag{1.8}$$

and the unit vector along **A** is given by

$$\mathbf{a}_A = \frac{A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \tag{1.9}$$

1.5 VECTOR ADDITION AND SUBTRACTION

Two vectors **A** and **B** can be added together to give another vector **C**; that is,

$$\mathbf{C} = \mathbf{A} + \mathbf{B} \tag{1.10}$$

The vector addition is carried out component by component. Thus, if $\mathbf{A} = (A_x, A_y, A_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$.

$$\mathbf{C} = (A_x + B_x)\mathbf{a}_x + (A_y + B_y)\mathbf{a}_y + (A_z + B_z)\mathbf{a}_z \tag{1.11}$$

Vector subtraction is similarly carried out as

$$\begin{aligned}\mathbf{D} &= \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) \\ &= (A_x - B_x)\mathbf{a}_x + (A_y - B_y)\mathbf{a}_y + (A_z - B_z)\mathbf{a}_z\end{aligned} \tag{1.12}$$

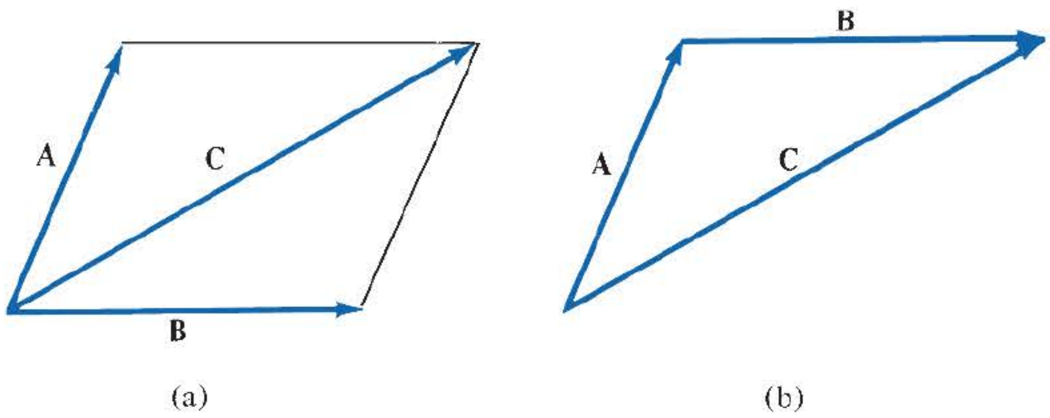

FIGURE 1.2 Vector addition $\mathbf{C} = \mathbf{A} + \mathbf{B}$: (**a**) parallelogram rule, (**b**) head-to-tail rule.

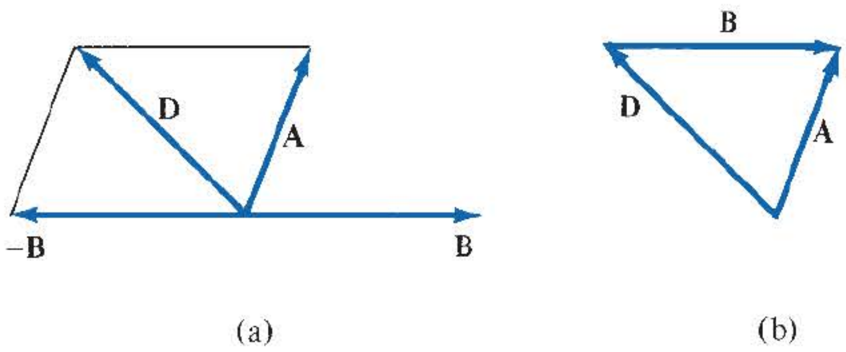

FIGURE 1.3 Vector subtraction $\mathbf{D} = \mathbf{A} - \mathbf{B}$: (**a**) parallelogram rule, (**b**) head-to-tail rule.

Graphically, vector addition and subtraction are obtained by either the parallelogram rule or the head-to-tail rule as portrayed in Figures 1.2 and 1.3, respectively.

The three basic laws of algebra obeyed by any given vectors **A**, **B**, and **C**, are summarized as follows:

Law	Addition	Multiplication
Commutative	$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$	$k\mathbf{A} = \mathbf{A}k$
Associative	$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$	$k(\ell\mathbf{A}) = (k\ell)\mathbf{A}$
Distributive	$k(\mathbf{A} + \mathbf{B}) = k\mathbf{A} + k\mathbf{B}$	

where k and ℓ are scalars. Multiplication of a vector with another vector will be discussed in Section 1.7.

1.6 POSITION AND DISTANCE VECTORS

A point P in Cartesian coordinates may be represented by (x, y, z).

The **position vector** $\mathbf{r}_P$ (or **radius vector**) of point P is defined as the directed distance from the origin O to P, that is,

$$\mathbf{r}_P = OP = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z \tag{1.13}$$

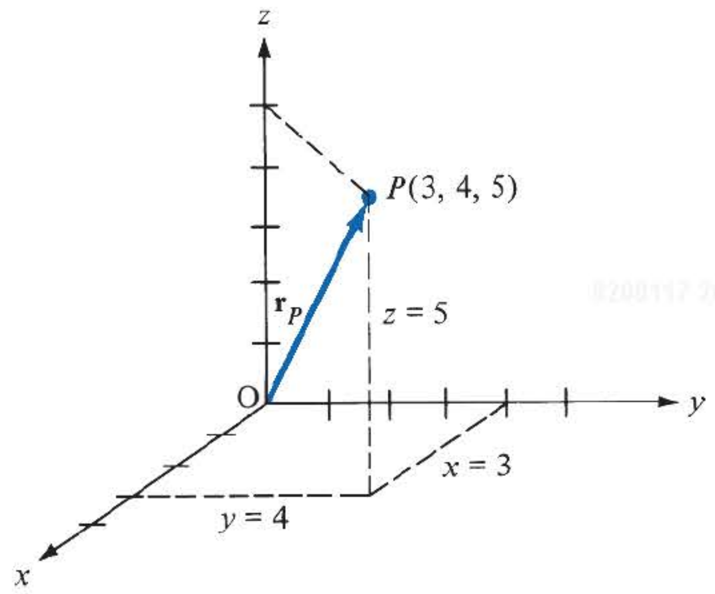

FIGURE 1.4 Illustration of position vector $\mathbf{r}_P = 3\mathbf{a}_x + 4\mathbf{a}_y = 5\mathbf{a}_z$.

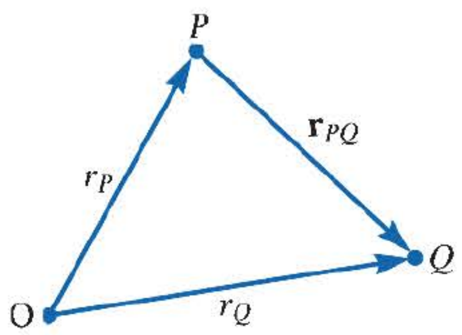

FIGURE 1.5 Distance vector $\mathbf{r}_{PQ}$.

The position vector of point P is useful in defining its position in space. Point (3, 4, 5), for example, and its position vector $3\mathbf{a}_x + 4\mathbf{a}_y + 5\mathbf{a}_z$ are shown in Figure 1.4.

The **distance vector** is the displacement from one point to another.

If two points P and Q are given by (x_P, y_P, z_P) and (x_Q, y_Q, z_Q), the *distance vector* (or *separation vector*) is the displacement from P to Q as shown in Figure 1.5; that is,

$$\begin{aligned} \mathbf{r}_{PQ} &= \mathbf{r}_Q - \mathbf{r}_P \\ &= (x_Q - x_P)\mathbf{a}_x + (y_Q - y_P)\mathbf{a}_y + (z_Q - z_P)\mathbf{a}_z \end{aligned} \tag{1.14}$$

The difference between a point P and a vector $\mathbf{A}$ should be noted. Though both P and $\mathbf{A}$ may be represented in the same manner as (x, y, z) and (A_x, A_y, A_z), respectively, the point P is not a vector; only its position vector $\mathbf{r}_P$ is a vector. Vector $\mathbf{A}$ may depend on point P, however. For example, if $\mathbf{A} = 2xy\mathbf{a}_x + y^2\mathbf{a}_y - xz^2\mathbf{a}_z$ and P is $(2, -1, 4)$, then $\mathbf{A}$ at P would be $-4\mathbf{a}_x + \mathbf{a}_y - 32\mathbf{a}_z$. A vector field is said to be *constant* or *uniform* if it does not depend on space variables x, y, and z. For example, vector $\mathbf{B} = 3\mathbf{a}_x - 2\mathbf{a}_y + 10\mathbf{a}_z$ is a uniform vector while vector $\mathbf{A} = 2xy\mathbf{a}_x + y^2\mathbf{a}_y - xz^2\mathbf{a}_z$ is not uniform because $\mathbf{B}$ is the same everywhere, whereas $\mathbf{A}$ varies from point to point.

EXAMPLE 1.1

If $\mathbf{A} = 10\mathbf{a}_x - 4\mathbf{a}_y + 6\mathbf{a}_z$ and $\mathbf{B} = 2\mathbf{a}_x + \mathbf{a}_y$, find (a) the component of $\mathbf{A}$ along $\mathbf{a}_y$, (b) the magnitude of $3\mathbf{A} - \mathbf{B}$, (c) a unit vector along $\mathbf{A} + 2\mathbf{B}$.